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# MODELING THE EFFECT OF COMPETITION ON TREE DIAMETER GROWTH AS APPLIED IN STEMS 

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Competition is a vital ingredient in individual-tree growth models. The advent of large computer-based tree-growth simulation systems has greatly increased the detail of competition that can be included. Many of the competition indices now available require the location of competing trees, costly data to acquire. However, many potential tree growth model users only have tree-diameters data. For them, the more generalized distance-independent tree growth model fulfills a real need. In this paper I present a systematic approach to modeling competition that does not require information about location of trees in a stand.

The competition model described is currently used in STEMS, a Stand and Tree Evaluation and Modeling System (Belcher et al. 1982). STEMS is a revision of an earlier North Central Station tree growth projection system that was a part of a Forest Resources Evaluation Program (FREP). A collection of papers detailing the development of the initial growth projection system has been published (USDA Forest Service 1979).

A basic premise in the design logic of these forest growth projection systems is that the growth of a tree is the product of its potential growth and a modifier of that potential due to competition (Leary and Holdaway 1979):
$\begin{gathered}\text { Annual change } \\ \text { in tree diameter }\end{gathered}=\left[\begin{array}{c}\text { potential yearly } \\ \text { d.b.h. growth }\end{array}\right] \cdot\left[\begin{array}{c}\text { fraction of the potential } \\ \text { growth actually achieved }\end{array}\right]$
Potential growth, the growth possible when trees are free of competition, was estimated using the growth attained by the most rapidly growing dominant and codominant trees (Hahn and Leary 1979). Diameter growth, diameter, and crown ratio from these trees along with their stand site index were used to develop the model for potential growth. A model for estimating potential tree diameter growth was developed for each major Lake States tree species.

The modifer function, which accounts for those competitive processes that reduce the potential growth to actual growth (Leary and Holdaway 1979), depends on the characteristics of the stand. This paper shows how this modifer was developed and how competition is handled in it.

## THE FOREST COMMUNITY

The modifier function mathematically represents the effect of the forest community on a single tree's growth. The following stand characteristics are important in describing this environment for any tree or group of trees:

1. Stand density is a measure of crowding, expressed in terms of stand basal area. The greater the density, the slower each tree grows. The modifier, a nonlinear function of basal area, approaches 1 for low levels of basal area and approaches 0 for high levels of basal area.
2. Stand structure is the distribution of tree diameters in a stand. Two measures of stand structure are absolute tree size and relative tree size as measured by diameter.
3. Stand species composition is the stand's makeup by species. In addition to the tree size effect within species, mixed species stands have the effects of interaction among species.

A modifier function should account for all three of these stand social characteristics.

## PREVIOUS MODEL

## Form

The original model was a stand component model. Each tree list was broken down into a maximum of four species groups and each species group was divided into two size classes. These divisions were
called components. Each stand component was projected as a whole, and later the component's growth was allocated to the individual trees. Speciesspecific coefficients were calibrated for trees growing under nearly pure conditions. However, the component approach also allowed for interactions between components of different species. The original modifier of the potential due to competition was (Leary and Holdaway 1979):
$1-\mathrm{e}^{-\beta(\mathrm{AD}) / \mathrm{BA}}$
where BA was the stand basal area in square feet per acre and $\beta$ (AD) was a function of the average diameter ( AD ) of the stand (fig. 1). This simple nonlinear function had the desirable property that the modifier approached 1 for low levels of basal areas and approached 0 at high basal areas.

The original form contained a complex expression for basal area. $\beta$ remained constant for any stand component of average diameter AD while the basal area of the component was altered via the concept of "effective basal area". A separate rule was used to allocate growth to the trees in each component according to its relative position in the component.

## Shortcomings

The stand component approach has several limitations. First, no data were available for determining the modifier coefficients for the less abundant species because the necessary pure ${ }^{1}$ stands for these species were not available.

Second, even for the abundant species it was almost impossible to find pure stands with very small or very large average diameters. This meant that projections made with the stand component model were only good for most major species over normal d.b.h. ranges. Outside of these ranges the model did not project growth well, especially for minor species and young stands. This could cause problems in growing regeneration stands, because these young stands would be far out of the range of the calibration data.

Finally, when the modifier (2) was tested on long range projections of red pine plantations, stand basal areas were obtained that exceeded known biological limits. The modifier remained high within very dense stands (fig. 1), especially for medium-sized overstory trees. These trees were projected to grow at 20 to 30

[^0]

Figure 1.-The behavior of the original modifier form in equation (2) and the initially proposed form in equation (3) for three different $\beta$ (or Bo) values.
percent of their potential though basal areas were as high as 400 to 500 square feet.

## PRESENT MODEL

## Development

Initially, an individual-tree model similar to the original stand component model was proposed:

MODIFIER $=1-e^{-B \alpha A D, R / B A}$.
Here Bo is a species-specific function of the stand average diameter (AD) and the ratio ( R ) of the tree's diameter to the stand's AD. In this model a tree's growth under competition depends on the stand's average tree size and the tree's relative position in the stand. Interactions among species are not included.

The basal area in equation (3) is constant for all trees in a stand. However, the Bo term is different for each individual tree (fig. 1). With this model, the higher the Bo value within a given stand the higher the proportion of potential achieved.

To better account for absolute and relative size effects, we replaced Bo in equation (3) by two separate multiplicative functions. Written in notational form:
$B o=f(R) g(A D)$.
This adjusts the stand's average diameter effect by each tree's relative position in the stand. No adjustment is made for a tree of mean stand diameter (i.e., $f(R=1)=1)$. If the tree is an "overstory" tree (one with a large relative diameter), the average diameter effect is increased accordingly; it is decreased for an "understory" tree (one with a small relative diameter).

From graphs of the data expressing Bo as a function of average diameter and relative diameter classes, appropriate functions were sought for $g(A D)$ and $f(R)$. A fairly simple two parameter function, $\mathrm{c}_{1}(\mathrm{AD}+1)^{c^{c 2}}$, used for $\mathrm{g}(\mathrm{AD})$ (fig. 2) fits the data well while allowing for monotonic decreasing curves that are evident on a few species. It does not fluctuate sharply up or down but maintains realistic behavior beyond the general range of the existing data.

For many species the relative function appears to be nearly linear over the central range of the data. However, the function appears to level off for data outside this central range, implying that both very small understory trees and very large overstory trees asymptotically approach limiting values. Thus a sigmoid function plus a constant, $b_{4}$, were used to estimate the relative effect (fig. 3). As a tree's relative position in the stand approaches zero, $f(R)$ will approach $b_{4}$. This provides a very small tree on an opengrown stand of large trees some minimal amount of growth that will permit it to struggle along.

Because of its shape, this modifier function still could seriously overgrow stands on long-term projections (fig. 1). Therefore we added a maximum stand basal area term that would limit tree size. Hence the modifier became:
MODIFIER $=1-\mathrm{e}^{\mathrm{Bd}(\text { (BAmax-BA)/BA }]^{p}}$.

When BA is greater than or equal to BAmax, the modifier equals zero. Because we had little data on species maximum basal areas, these values were set empirically.

The power p on the basal area term adds flexibility to the resulting curves. The value of 0.5 was chosen


Figure 2.-Average diameter effect $g(A D)$ for selected Lake States tree species.


Figure 3.-Relative diameter effect $f(R)$ for selected Lake States tree species.
based on the comparison of graphs of equation (5) for p values of $0.5,1.0$, and 2.0 with graphs of the actual data. The final form of the modifier becomes:

$$
\begin{equation*}
\text { MODIFIER }=1-\mathrm{e}^{-\mathrm{Bd}(\mathrm{BAmax}-\mathrm{BA}) / \mathrm{BA})^{0.5}} . \tag{6}
\end{equation*}
$$

where $B o=f(R) \cdot g(A D)$.
Recall that the effect of the surrounding forest community on a tree or group of trees is described by stand density, structure (i.e., actual and relative tree size), and species composition (including species interaction). Equation (6) accounts for all of these except species interaction. From preliminary model testing it appeared that for most species the relative diameter effect was the most important, followed by the average diameter effect. The species interaction effect was usually small so was not included in the model.

## Final Form

The final form of the model is:

$$
\begin{gather*}
\text { MODIFIER }=1-e^{-f(R) g(A D)(B A \max -B A) / B A]^{5}}  \tag{7}\\
\text { where } f(R)=b_{1}\left[1-e^{b 2 R}\right]^{b_{3}}+b_{4},  \tag{8}\\
g(A D)=c_{1}(A D+1)^{c_{2}}, \tag{9}
\end{gather*}
$$

and $\mathrm{b}_{1}, \mathrm{~b}_{2}, \mathrm{~b}_{3}, \mathrm{~b}_{4}, \mathrm{c}_{1}$, and $\mathrm{c}_{2}$ are unknown numerical constants.

To avoid computing implausible values for the parameters, the following restrictions were made: $b_{1} \leqslant 5, b_{4} \geqslant 0, c_{1}>0$, and $c_{2} \leqslant 1$. In equation ( 8 ), $b_{1}+b_{4}$ is the asymptotic maximum of the relative diameter effect, $b_{4}$ is its minimum value, $b_{2}$ is a rate parameter,
and $b_{3}$ is a rate and shape parameter. In expression (9), $c_{1}$ is the $y$-intercept of the average diameter effect and $c_{2}$ provides a measure of its slope. When $c_{2}$ equals 1.0 , the function is linear. When $\mathrm{c}_{2}$ is negative, the average diameter effect is decreasing with increasing diameter.

## ESTIMATING COEFFICIENTS

## Data Base

Now that the form for the modifer was set, we had to determine species-specific constants for it. To do this we first had to calculate potential growth from the measured d.b.h., crown ratio, and plot site index. Then using this known potential growth, as well as the measured realized growth, we could estimate the unknown coefficients in the modifier function.

Certain restrictions were imposed on the data before tree observations could be used to derive the constants. To be included in the study a tree needed to have survived over the time interval chosen, to have a measured crown ratio and plot site index to obtain the growth potential, and to have a measured diameter at the beginning and end of the interval so growth could be determined for the period. Two growth intervals were selected on each plot to obtain a maximum of two observations per tree: one from the final measurement back approximately 10 years, and the other from the first measurement forward approximately 10 years. In this way fuller use was made of the data base because frequently trees classified as ingrowth, dead, or cut had survived through one of the two intervals. If the two measurements overlapped, only the last interval was used.

We only included plots that had no major disturbance (i.e., losses to excessive mortality or to cutting) during the growth interval. For example, if the final basal area was less than 90 percent of the initial basal area, we assumed that a major disturbance occurred and did not use the plot.

Permanent growth plot data were available from 44 different studies in Minnesota, Wisconsin, and Michigan (Christensen et al. 1979). There were 1,501 plots containing 92,649 trees remeasured at least once. From this, a total of 72,923 tree observations
were obtained. Because previous analyses showed that red pine in natural stands and plantations grow differently (Leary et al. 1979), we separated the data for natural-grown and plantation-grown red pine. We did not separate the data for any other species.

## Methods

For each tree we calculated the growth potential using the equation and coefficients as described in Hahn and Leary (1979). We calculated the modifier, the proportion of potential growth achieved, using

$$
\underset{\left(\begin{array}{c}
\text { proportion of potential growth }  \tag{10}\\
\text { (i.e. modifier) }
\end{array}\right.}{=\frac{\text { actual growth }}{\text { calculated potential growth }} .}
$$

Trees were grouped on the basis of stand basal area (BA), stand average diameter (AD), and their relative position in the stand (R)-all variables in the modifier function. Cells were formed for each species using 2 inch average diameter classes, 25 square feet basal area classes, and 10 percent relative diameter classes. The mean MODIFER was computed for each cell.

For each species these cell modifier observations were graphed for basal area levels coded by relative diameter class. Species with a large data base showed distinct trends consistent with the general form of the function in equation (6). The logical maximum basal area approached by each species regardless of site was determined from these graphs. These maximums were checked against expectations obtained from the scientific judgment of foresters before being used.

All six parameters in equations (8) and (9) can be estimated simultaneously, but the natural condition where $R=1$ doesn't necessarily yield $f(1)=1$. This makes it difficult to compare the relative effect among species. This problem can be avoided by fitting the model in two steps as follows: first, estimate the coefficients in the $g(A D)$ function setting $f(R)$ to 1 ; then substitute the $c_{1}$ and $c_{2}$ coefficients of equation (9) into equation (7) and estimate the four parameters in $f(R)$, equation (8). With this procedure $f(R)$ will approximate 1 for $\mathrm{R}=1$.

## RESULTS OF ANALYSIS

Using the modifier model, we computed coefficients for 26 Lake States species (table 1, figs. 2 and 3).

Table 1.-Species coefficients for modifier function ${ }^{1}$ for major Lake States tree species

| Species | Observations | BAmax | $\mathrm{c}_{1}$ | $\mathrm{C}_{2}$ | $\mathrm{b}_{1}$ | $\mathrm{b}_{2}$ | $\mathrm{b}_{3}$ | $b_{4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Number | $\mathrm{Ft}^{2} / \mathrm{ac}$ |  |  |  |  |  |  |
| Jack pine | 8,934 | 225 | 0.402 | 0.230 | 1.78 | $-3.00$ | 16.20 0 | 0.227 |
| Red pine- ${ }^{2}$ | 6,067 | 300 | 2.030 | -. 354 | . 72 | -10.90 | 1,688.30 | 375 |
| Red pine- $\mathrm{P}^{2}$ | 20,224 | 350 | . 441 | . 173 | 2.31 | -1.67 | 3.94 | . 000 |
| White pine | 1,162 | 300 | . 097 | . 755 | 1.36 | -2.64 | 11.50 | . 386 |
| White spruce | 2,440 | 350 | 1.507 | -. 520 | 5.00 | -1.01 | 3.64 | . 000 |
| Balsam fir | 4,376 | 325 | . 927 | -. 299 | 1.76 | -1.51 | 2.63 | . 233 |
| Black spruce | 2,870 | 300 | . 522 | . 173 | 3.80 | -1.52 | 6.54 | . 348 |
| Tamarack | 57 | 250 | . 039 | 1.000 |  | Use jack pine ${ }^{3}$ |  |  |
| N. white-cedar | 2,782 | 350 | . 526 | . 136 | 2.54 | -1.14 | 2.26 | . 000 |
| Hemlock | 334 | 300 | . 046 | 1.000 | 1.27 | -1.34 | 1.05 | . 000 |
| Black ash | 174 | 250 | . 260 | .419 | 5.00 | -. 57 | 1.83 | . 063 |
| Cottonwood | Use red maple ${ }^{4}$ |  |  |  |  |  |  |  |
| Silver maple | Use red maple ${ }^{4}$ |  |  |  |  |  |  |  |
| Red maple | 1,514 | 250 | . 181 | . 445 | 1.40 | -2.03 | 10.40 | . 694 |
| Elm | 1,585 | 250 | . 100 | . 629 | 5.00 | -. 97 | 4.40 | . 268 |
| Yellow birch | 656 | 250 | . 202 | . 454 | . 68 | -10.97 | 1,568.20 | . 483 |
| Basswood | 2,096 | 250 | . 353 | . 182 | 1.59 | -3.27 | 26.70 | . 412 |
| Sugar maple | 9,383 | 250 | . 142 | . 524 | 1.17 | -4.59 | 29.19 | . 430 |
| White ash | 1,500 | 250 | . 453 | . 340 | 5.00 | -1.38 | 8.26 | . 326 |
| White oak | 623 | 250 | . 051 | 1.000 |  | Use select red oak ${ }^{3}$ |  |  |
| Select red oak | 1,307 | 275 | . 278 | . 365 | 1.98 | -. 97 | 1.64 | . 000 |
| Other red oak | 80 | 250 | 1.365 | -. 208 |  | Use select red oak ${ }^{3}$ |  |  |
| Hickory | 507 | 250 | . 280 | . 228 | 1.66 | -2.62 | 9.97 | . 515 |
| Bigtooth aspen | 94 | 250 | . 093 | 1.000 | 1.13 | -4.64 | 164.62 | . 648 |
| Quaking aspen | 2,944 | 250 | . 209 | . 543 | 1.08 | -6.60 | 346.09 | . 395 |
| Paper birch | 1,214 | 275 | . 110 | . 678 | 1.98 | -1.75 | 3.67 | . 232 |

${ }^{1}$ MODIFIER $=1-\mathrm{e}^{-(\mathrm{R}) \mathrm{g})(\mathrm{ADFf}(\mathrm{BAmax}-\mathrm{BA}) / \mathrm{BA}]^{5}}$
where $f(R)=b_{1}\left[1-e^{b_{2} R}\right]^{b_{3}}+b_{4}$, and

$$
g(\mathrm{AD})=\mathrm{c}_{1}(\mathrm{AD}+1)^{\mathrm{c} 2}
$$

${ }^{2} P=$ plantation
$N=$ natural stand .
${ }^{3}$ Use coefficients for similar species because data for these species were insufficient to reliably estimate the relative diameter effect.
${ }^{4}$ Use red maple coefficients because no data were available for these species.

## Illustration

To demonstrate graphically how the modifier function behaves, consider a select red oak stand with mean stand diameter of 10 inches. The model is used to evaluate the growth of five trees having diameters of $4,7,10,13$, and 16 inches. The corresponding relative diameter values ( $R$ ) of these trees are $0.4,0.7$, $1.0,1.3$, and 1.6. Using a series of species-specific
curves for these five $R$ values (fig. 4), $\mathrm{Bo}(\mathrm{f}(\mathrm{R}) \mathrm{g}(\mathrm{AD})$ can be estimated given any $A D$ and $R$ values. Incorporating these five Bo values into the modifier function in equation (6) yields figure 5.
In a stand with low basal area, $\mathrm{BA}_{1}$, the large trees will grow at nearly full potential but the understory trees will not-their growth will be somewhat restricted. In a stand with high basal area, $\mathrm{BA}_{2}$, the overstory trees will grow a little, even under these


Figure 4.-Various relative diameter curves added onto the basic average diameter effect curve for select red oak. For any given $A D$ and $R$ values Bo can be found. The five points indicate Bo values for the trees in the example.
adverse conditions, but growth of the understory trees will be severely restricted.

The same type of results can be shown in a different form using three-dimensional graphs. Graphs of the modifier function for sugar maple at basal areas of 75 and 150 sq . ft ./acre indicate how the effects of average diameter and relative diameter combine to influence the proportion of the potential growth realized (fig. 6).


Figure 5.-Modifier function for five Bo values corresponding to relative diameters of $0.4,0.7,1.0,1.3$, and 1.6 within a select red oak stand having a 10 inch average stand diameter at high and at low basal area.


Figure 6.-The modifier function for sugar maple for stand basal areas of 75 and 150 sq. ft./acre.

## Biological Considerations

What happens as an untreated stand approaches the maximum basal area? Overstory trees will still be given some growth whereas understory trees will approach zero growth. In STEMS, as the growth rate decreases the probability of death increases. This reduces the basal area and releases the larger trees slightly. Therefore, the stand will approach an equilibrium just below the maximum basal area. We suspect that for some species maximum basal area is
related to site, with better sites having higher maximum basal area. But we did not have the data to test this.
Gingrich (1967) has shown that stands with large average diameter can support more basal area without adversely affecting the growth of the individual tree. The average diameter effect for most species confirms this (fig. 2). For example, a 10 inch tree in a stand with mean d.b.h. 10 inches will have greater growth than a 4 inch tree in a 4 inch stand, assuming equal stand basal area. This seems to be a reasonable result. That, however, does not account for those species with decreasing average diameter effects (e.g., white spruce in fig. 2). These species may have faulty growth potential functions (calibrated from a limited range of data) and the modifier function may be forced to compensate for these errors.

What happens if a previously set maximum basal area (table 1) doesn't seem appropriate for a particular forest condition? In theory, the data should be refit and coefficients determined for the new BAMAX.

In practice, if you are willing to accept minimal side effects, BAMAX can be adjusted without refitting the data and changing the coefficients. To illustrate, a minimal change in BAMAX of 25 square feet only slightly alters the value of the modifier function in equation (7). For the extreme change of increasing BAMAX from 275 to 350 square feet (fig. 5), the modifier for all trees would be increased by 0.02 to 0.10 on plots of average density, say 100 to 150 square feet. The change in the proportion of growth would be greater than 0.10 for plot BA's close to the new BAMAX. Likewise, a similar decrease in BAMAX produces corresponding decreases in the modifier. Adjusting BAMAX, which produces slight increases or decreases in the growth projections, enables users to account more accurately for known local conditions or to include the influence of good or poor sites.

## EVALUATION

To evaluate model performance, we used the STEMS system containing the modifier function (7) on two sets of data. The first was a systematic sample of every fifth plot from the calibration data base. Any marked bias at this point would indicate a basic flaw in the mathematical model used to describe the system.

The projection system can perform well over the broad region from which the calibration data were drawn and still fail when representing growth of smaller subregions. Therefore, we also tested the
model against several independent data sets (Schaeffer 1980 and Taylor 1979). This second data set (or validation data base) consisted of five independent data sources from the Lake States region (Holdaway and Brand 1983).
We also tested the new model against the best available previous model, equation (2). The same growth potential function was used in both models. Crown ratio is a variable in the potential function. To avoid introducing a crown ratio error, we only used trees with measured crown ratios.
The growth for each tree on the test plots that had survived at the last measurement was projected for the preceding time interval. Tree and stand characteristics at the first measurement were used as initial conditions for the projection. Cut trees were removed and ingrowth trees added to the projection tree list in the appropriate year. Final predicted and observed d.b.h.'s, along with the pertinent stand and tree data, were recorded. Then we calculated each tree's d.b.h. error (i.e., predicted minus observed d.b.h.).
The projection intervals ranged from 9 years to 17 years. The errors were standardized to 10 years using the adjustment:

$$
\begin{equation*}
\text { (predicted d.b.h. - observed d.b.h) } \quad \text { X } 10 \tag{11}
\end{equation*}
$$

number of years in measurement interval
A positive error means growth was overpredicted.
We analyzed the results for accuracy and precision by calculating the mean and standard deviation of the 10 year errors (table 2). If the predictions were perfect, all errors would be zero and both the mean and standard deviation would be zero. The further these values are from zero, the greater the bias and variability.

We also investigated species interaction. For the validation data, the mean d.b.h. error for each species was broken down by the forest type in which it occurred. These results help answer questions such as:

Table 2.-Summary of the mean and standard deviation of the d.b.h. errors in 10 years for the two models

| D.b.h. errors <br> (10 years) | Calibration |  |  | Validation |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
|  | Original | New |  | Original | New |
| Mean | 0.02 | -0.03 |  | 0.13 | 0.11 |
| Standard deviation | .52 | .46 |  | .73 | .63 |
| Number of plots | 293 |  | 822 |  |  |
| Number of trees ${ }^{1}$ | 7,702 |  | 11,182 |  |  |

[^1]Do trees of a given species grow the same in mixed stands as in pure stands or in one forest type as in another? The results for the original model (with an interaction term) has 14 species-forest type combinations representing 477 trees showing definite interactions between trees of one species growing on a different type forest. The STEMS model with no interaction term (7), has only 6 problem combinations for a total of 199 trees out of 11,182 trees. This represents only 1.8 percent of the trees on the validation data base. Hence the STEMS model, even without attempting to account for species interaction, handles it well.

## DISCUSSION

Our test results show that one could justify choosing either model because neither is clearly superior (table 2). An indepth evaluation of both models showing strengths and weaknesses is given in Holdaway and Brand (1983). However, the STEMS model does somewhat better in three of the four performance tests. Furthermore, the STEMS model was judged to be clearly preferable on the basis of ease in

1. calibrating and recalibrating the model,
2. understanding the various components of the model,
3. programming the model and understanding the projection program, and
4. adjusting the model for specific user needs.

The original model had some complex parts that were hard to calibrate, understand, or adjust. As Schaeffer (1980) points out, there are tradeoffs in choosing one model over another and "the simplest model which can be acceptably validated is deemed more suitable than the more complex model". Taking all of the above factors into consideration, the new model was recommended and has been included in the newest STEMS system.

## CONCLUSION

The realized growth of a tree in a stand of trees can be characterized by the product of a potential growth function and a modifier of the potential. The modifier function (7) we devised adequately describes how this potential growth is reduced due to the social characteristics of a stand. The $[(\mathrm{BAmax}-\mathrm{BA}) / \mathrm{BA}]^{5}$ term provides for the effect of crowding (or competition). The modifier becomes zero when the maximum basal area (BAmax) is reached for a species. The $\mathbf{g}$ function weights the basal area term to account for the stand effect of the average diameter and the function then
adjusts the index according to the relative diameter of each tree in the stand. Because Bo is a species-specific variable, each species in the stand is grown separately.

Our proposed model performs well for a wide range of Lake States species. It does slightly better than the earlier stand component modifier and is also a simpler, more flexible model.

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The modifier function used in STEMS (Stand and Tree Evaluation and Modeling System) mathematically represents the effect that the surrounding forest community has on the growth of an individual tree. This paper 1) develops the most recent modifier function, 2) discusses its form, 3) reports the results of the analysis with biological considerations, and 4) evaluates the performance of this new model.

KEY WORDS: growth model, competition, projection system, forest growth, Lake States species



[^0]:    ${ }^{1} A$ stand is considered "pure" if more than 75 percent of the stand is one species.

[^1]:    ${ }^{1}$ Value for STEMS model.

