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A COMPUTER PROGRAM FOR MAKING S-CONTRASTS INVOLVING LINEAR COMBINATIONS OF GROUP MEANS

Abstract. A description of a FORTRAN IV computer program that is used for making Scheffé's S-contrasts in one-way analyses.

Forestry research often involves experiments in which many treatments are tested. If the overall test of treatment equality results in a significant difference, it is usually necessary to make further tests involving linear combinations of the treatment responses. When many a *posteriori* comparisons are necessary and treatments have unequal sample sizes, the researcher often resorts to Scheffé's S-contrasts and finds himself involved in lengthy calculations.

Scheffé¹ describes the method of making contrasts for a one-way anal-

ysis. In short, the method results in the contrast $\sum_{i=1}^I c_i \beta_i$ with a confidence

interval:

$$\sum_{i=1}^I c_i \beta_i \pm (I-1) (F_{\alpha; I-1, N-I}) (s) \sqrt{\sum_{i=1}^I (c_i^2 / J_i)}$$

Where:

c_i = a contrast multiplier for group i , and $\sum_{i=1}^I c_i = 0$.

β_i = arithmetic mean of group i .

¹Scheffé, Henry. THE ANALYSIS OF VARIANCE. John Wiley, N. Y. 477 pp. 1959.

I = number of groups.

F = Fisher's F-value for I-1 and N-I degrees of freedom at the chosen (α) level of significance.

N = total sample size (all groups combined).

s^2 = the error mean square.

J_i = sample size for group i.

By the same method, we may calculate an F-value for an individual contrast as:

$$F_{I-1, N-I} = \frac{\left(\begin{array}{c} I \\ \sum c_i \beta_i \\ 1 \end{array} \right)^2}{(I-1) s^2 \left(\begin{array}{c} I \\ \sum c_i^2 / J_i \\ 1 \end{array} \right)}$$

and we may check the F-value against a tabulated F at level of significance α .

The FORTRAN IV computer program described here is simply an automated method of making one-way analysis S-contracts for linear combinations of group means with constant or varying sample sizes.

Description of Control Deck

The user must supply the following control deck as data to be operated on by the program:

<i>Card No.</i>	<i>Card Columns</i>	<i>Content</i>
1	1-4	Number of treatments or groups, right adjusted.
	5-8	Degrees of freedom for error, right adjusted.
	9-17	Error mean square, punched with decimal point.
	18-22	MEANS punched if individual treatment means are to be compared; blank otherwise.
2-I	1-10	Label for group.
(1 for each group)	11-13	Sample size, right adjusted
	14-20	Group total, punched with decimal point.
	21	Minus sign (—) if multiplier (c) for contrast is negative.

22-25 Multiplier (c) for contrast,
punched with decimal point.
26-70 Repetition of format for columns
21-25 for each additional contrast
to be made.

The program is limited to:

- (1) A maximum of 10 contrasts (in addition to the contrasts of individual means).
- (2) A maximum of 1,000 groups.

Example

If we assume that we have tested the effects of eight fertilizers on height growth of red pine seedlings and that the treatments are represented by unequal sample sizes, then the analysis might be:

Source	df.	SS	M.S.	F.
Fertilizer	7	94.0678	13.438	14.0
Error	134	128.7060	0.960	
Total	141	222.7738		

The treatment totals and their sample sizes are:

Treatment	J_i	Total (growth in height)
1	18	36.7
2	19	12.1
3	17	11.8
4	15	42.1
5	19	38.9
6	20	60.3
7	20	40.2
8	14	30.1

	8	134	0.960	MEANS		
TREAT. 1	18	36.7		.25	.25	
TREAT. 2	19	12.1	.5	-.5		
TREAT. 3	17	11.8	.5	-.5		
TREAT. 4	15	42.1	-.5		-.5	
TREAT. 5	19	38.9		.25	.25	
TREAT. 6	20	60.3	-.5		-.5	
TREAT. 7	20	40.2		.25	.25	
TREAT. 8	14	30.1		.25	.25	

Figure 1.—Control deck.

S CONTRASTS INVOLVING LINEAR COMBINATIONS OF ARITHMETIC MEANS -
 EACH CONTRAST HAVING 7 AND 134 DEGREES OF FREEDOM

TREAT. 1 VS. TREAT. 2 , F= 0.2704E 01
 TREAT. 1 VS. TREAT. 3 , F= 0.2353E 01
 TREAT. 1 VS. TREAT. 4 , F= 0.7177E 00
 TREAT. 1 VS. TREAT. 5 , F= 0.9890E-04
 TREAT. 1 VS. TREAT. 6 , F= 0.1343E 01
 TREAT. 1 VS. TREAT. 7 , F= 0.1177E-02
 TREAT. 1 VS. TREAT. 8 , F= 0.1447E-01
 TREAT. 2 VS. TREAT. 3 , F= 0.4380E-02
 TREAT. 2 VS. TREAT. 4 , F= 0.5873E 01
 TREAT. 2 VS. TREAT. 5 , F= 0.2813E 01
 TREAT. 2 VS. TREAT. 6 , F= 0.8200E 01
 TREAT. 2 VS. TREAT. 7 , F= 0.2734E 01
 TREAT. 2 VS. TREAT. 8 , F= 0.2746E 01
 TREAT. 3 VS. TREAT. 4 , F= 0.5292E 01
 TREAT. 3 VS. TREAT. 5 , F= 0.2445E 01
 TREAT. 3 VS. TREAT. 6 , F= 0.7366E 01
 TREAT. 3 VS. TREAT. 7 , F= 0.2368E 01
 TREAT. 3 VS. TREAT. 8 , F= 0.2422E 01
 TREAT. 4 VS. TREAT. 5 , F= 0.7192E 00
 TREAT. 4 VS. TREAT. 6 , F= 0.5536E-01
 TREAT. 4 VS. TREAT. 7 , F= 0.8095E 00
 TREAT. 4 VS. TREAT. 8 , F= 0.4647E 00
 TREAT. 5 VS. TREAT. 6 , F= 0.1358E 01
 TREAT. 5 VS. TREAT. 7 , F= 0.2025E-02
 TREAT. 5 VS. TREAT. 8 , F= 0.1263E-01
 TREAT. 6 VS. TREAT. 7 , F= 0.1503E 01
 TREAT. 6 VS. TREAT. 8 , F= 0.9169E 00
 TREAT. 7 VS. TREAT. 8 , F= 0.2402E-01

0.5000E 00 X TREAT. 2
 0.5000E 00 X TREAT. 3
 -0.5000E 00 X TREAT. 4
 -0.5000E 00 X TREAT. 6
 F= 0.1316E 02

0.2500E 00 X TREAT. 1
 -0.5000E 00 X TREAT. 2
 -0.5000E 00 X TREAT. 3
 0.2500E 00 X TREAT. 5
 0.2500E 00 X TREAT. 7
 0.2500E 00 X TREAT. 8
 F= 0.6871E 01

0.2500E 00 X TREAT. 1
 -0.5000E 00 X TREAT. 4
 0.2500E 00 X TREAT. 5
 -0.5000E 00 X TREAT. 6
 0.2500E 00 X TREAT. 7
 0.2500E 00 X TREAT. 8
 F= 0.2466E 01

Figure 2.—Program output.

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$IBFTC STEST
  DIMENSION LABEL(1000,3),XJ(1000),TOT(1000),C(1000,10)
  READ(5,1)IGRPS,IDF,SQUARE,MEAN,S
1  FORMAT(2I4,F9.0,A4,A1)
  DATA MEA,SN/4HMEAN,1HS/
  XGRPS=IGRPS
  DF=IDF
  IGRPDF=IGRPS-1
  DO 2 I=1,IGRPS
2  READ(5,3)(LABEL(I,J),J=1,3),XJ(I),TOT(I),(C(I,J),J=1,70)
3  FORMAT(2A4,A2,F3.0,F7.0,10F5.0)
  WRITE(6,4)IGRPDF,IDF
4  FORMAT(85H15 CONTRASTS INVOLVING LINEAR COMBINATIONS OF ARITHMETIC
1  MEANS - EACH CONTRAST HAVING15,4H AND15,20H DEGREES OF FREEDOM )
  IF(MEA.NE.MEAN.OR.SN.NE.S)GO TO 7
  DO 5 I=1,IGRPDF
  IZ=I+1
  DO 5 K=IZ,IGRPS
  DIFF=TOT(I)/XJ(I)-TOT(K)/XJ(K)
  SUM=1./XJ(I)+1./XJ(K)
  F=(DIFF**2)/((XGRPS-1.)*SQUARE*SUM)
5  WRITE(6,6)(LABEL(I,J),J=1,3),(LABEL(K,J),J=1,3),F
6  FORMAT(2H0 2A4,A2,5H VS. 2A4,A2,4H. F=E11.4)
7  DO 13 J=1,10
  WRITE(6,8)
8  FORMAT(1H )
  DO 9 I=1,IGRPS
  IF(C(I,J).NE.0.0)GO TO 10
9  CONTINUE
  GO TO 15
10 DIFF=0.
  SUM=0.
  DO 12 I=1,IGRPS
  DIFF=DIFF+C(I,J)*(TOT(I)/XJ(I))
  IF(C(I,J).NE.0.0)WRITE(6,11)C(I,J),(LABEL(I,K),K=1,3)
11 FORMAT(1H E11.4,3H X 2A4,A2)
12 SUM=SUM+(C(I,J)**2)/XJ(I)
  F=(DIFF**2)/((XGRPS-1.)*SQUARE*SUM)
13 WRITE(6,14)F
14 FORMAT(1H 21X,2HF=E11.4)
15 STOP
  END

```

Figure 3.—Program listing.

Now, suppose that we want to make contrasts of:

- (1) individual treatment response means.
- (2) average of responses to treatments 2 and 3 versus average of 4 and 5.
- (3) average of responses to treatments 1, 5, 7, and 8 versus average of 2 and 3.
- (4) average of responses to treatments 1, 5, 7, and 8 versus average of 4 and 6.

Then the control deck for this job would be that shown in figure 1. The results are shown in figure 2. If we check the calculated F-values² against tabulated $F_{(.05)7,134} = 2.08$, we find the following contrasts to be significant:

- (1) Treatments 2 and 3, taken separately, versus each of the remaining treatment means (1,4-8).
- (2) Contrasts (2), (3), and (4), as stated above.

Figure 3 contains a program listing.

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²Note that the floating-point multipliers and F-values contain a decimal value, and, after the E, the number of places to the left (—) or right that the decimal point must be moved.

³When this paper was prepared, Dr. Frayer was a research forester with the Northeastern Forest Experiment Station, Forest Service, U. S. Department of Agriculture, Upper Darby, Pa.